Spread Spectrum Communications

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Basic concepts of digital communication

- **Probability of Error (P_E)**
- A measure of how well the demodulator and decoder perform is the frequency with which errors occur in the decoded sequence.
- More precisely, the average probability of a bit-error at the output of the decoder is a measure of the performance of the demodulator–decoder combination.
- In general, the probability of error is a function of
  - ➢the code characteristics,
  - ➢the types of waveforms used to transmit the information over the channel,
  - ➢the transmitter power,
  - ➢the characteristics of the channel i.e., the amount of noise, the nature of interference, etc.,
  - ➢and the method of demodulation and decoding.

Basic concepts of digital communication

- **Orthogonal and antipodal (polar) signals**
- Two signals $x_1(t)$ and $x_2(t)$ are said to be orthogonal if

$$\langle x_1(t), x_2(t) \rangle = \int_a^b x_1(t) x_2^*(t)\,dt = 0$$

- The integral above is referred to as the correlation between the two signals.
- Thus two signals are orthogonal when their correlation is zero.
- Probability of error of orthogonal signals:

$$P_b = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Basic concepts of digital communication

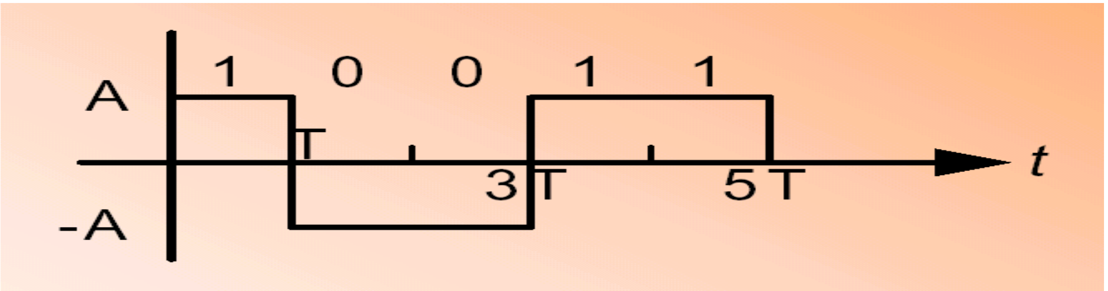
- **Orthogonal and antipodal (polar) signals**
- If two signals $s_1(t)$ and $s_2(t)$ satisfy the condition:

$$s_2(t) = -s_1(t)$$

signals are said to be antipodal or polar.

- Example

$$s_1(t) = A, \quad 0 \le t \le T, \quad \text{for binary } 1$$
$$s_0(t) = -A, \quad 0 \le t \le T, \quad \text{for binary } 0$$



Basic concepts of digital communication

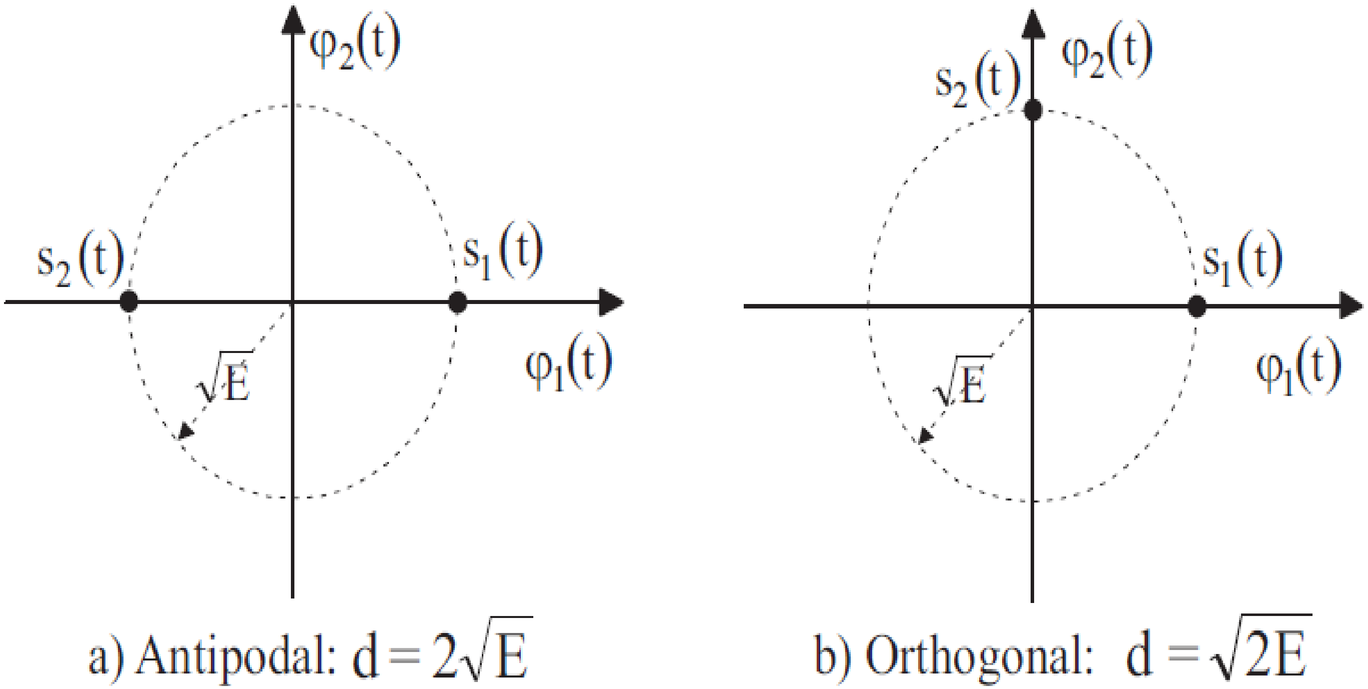
- **Orthogonal and antipodal (polar) signals**
- Probability of error of antipodal signals

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

- Bipolar signals require a factor of 2 increase in energy compared to Orthogonal signals.
- Since $10\ log_{10} 2 = 3\ dB$, we say that bipolar signaling offers a *3 dB* better performance than Orthogonal signaling.

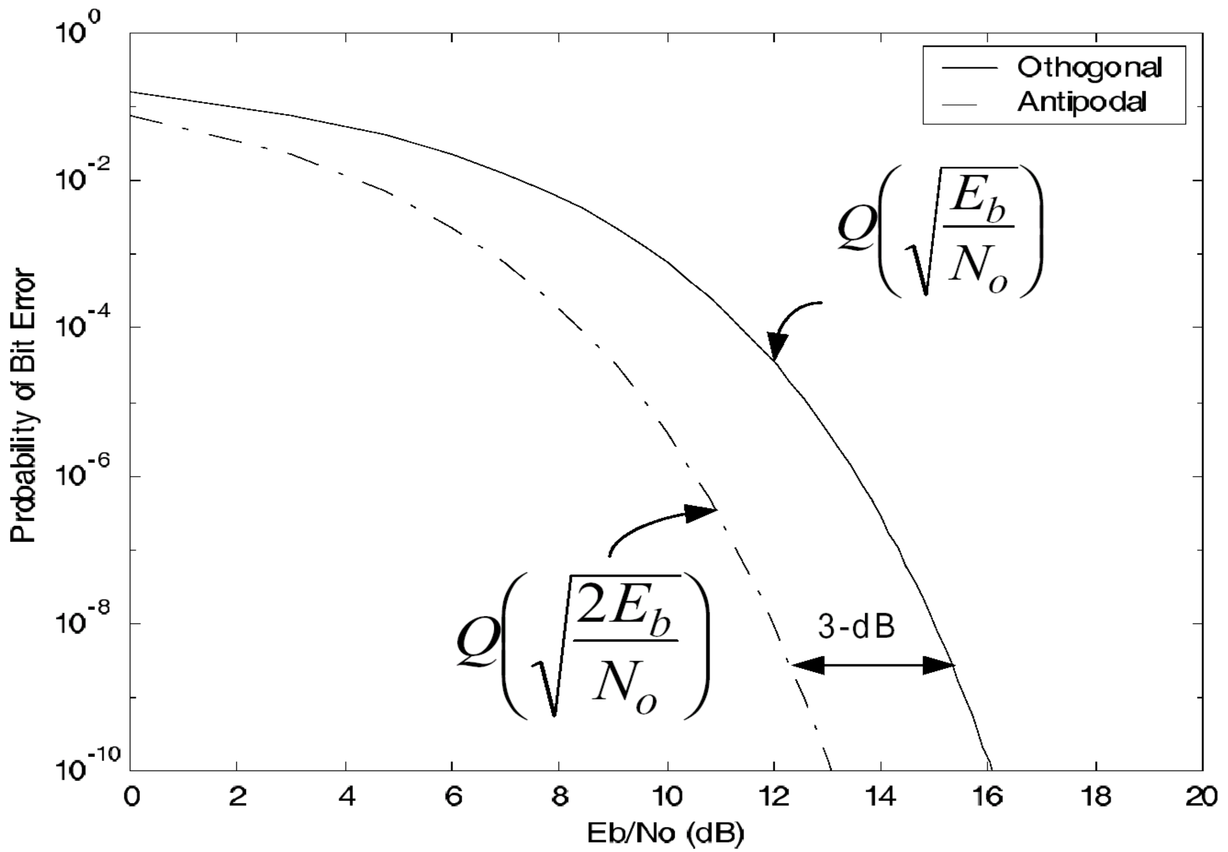
Basic concepts of digital communication

•Orthogonal and antipodal (polar) signals



Basic concepts of digital communication

- **Orthogonal and antipodal (polar) signals**



Basic concepts of digital communication

- **Q function**
- **Normal or Gaussian distribution**
- This distribution has two parameters, mean (μ) and variance ($\sigma^2 > 0$).
- The density of x (a random variable) is

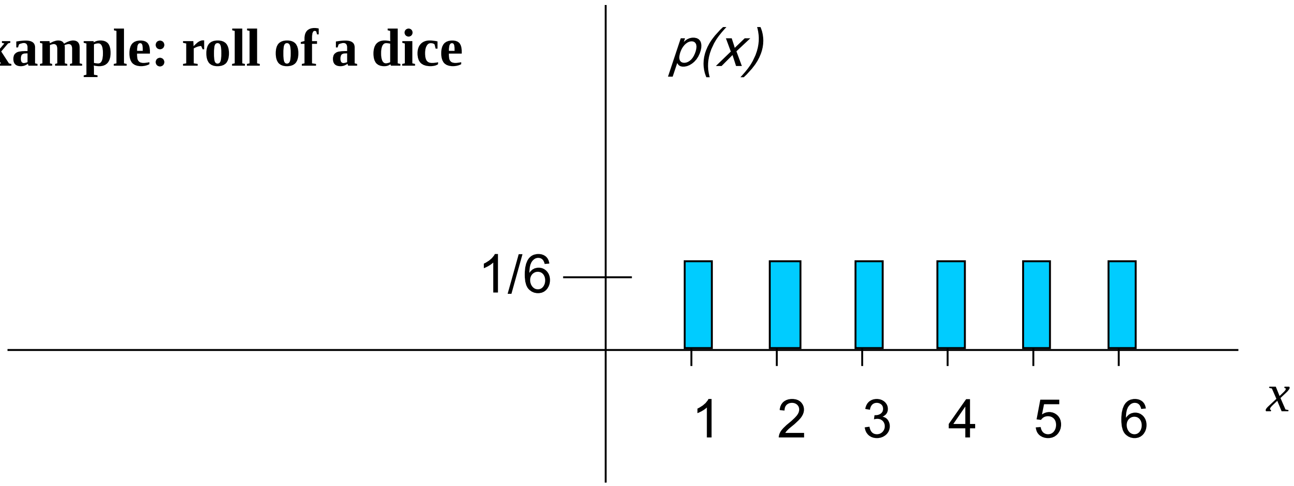
$$\phi\left(x:\mu,\sigma^2\right)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$$

- and the distribution function is

$$\Phi\left(x:\mu,\sigma^2\right)=\int_{-\infty}^{x}\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2\sigma^2}(t-\mu)^2}dt$$

Basic concepts of digital communication

- **Probability Distribution**
- A probability function maps the possible values of random variable *x* against their respective probabilities of occurrence, *p*(*x*).
- **Example: roll of a dice**

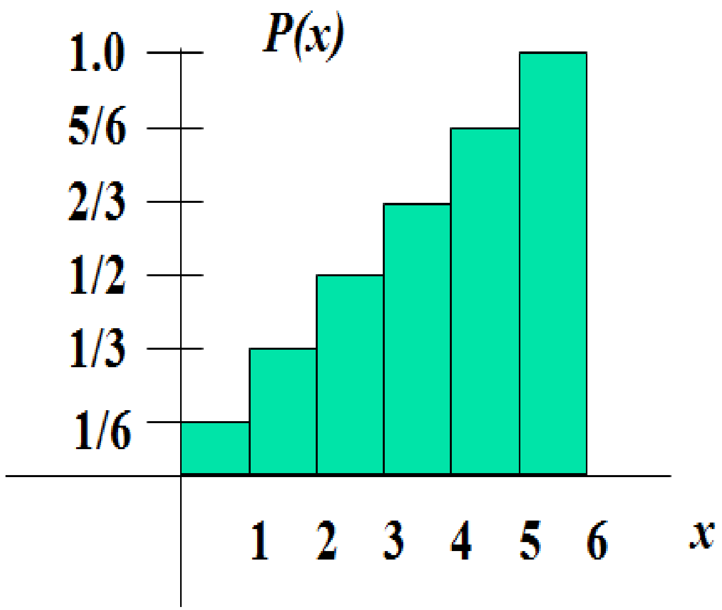


$$\sum_{\text{all } x} p(x) = 1$$

Basic concepts of digital communication

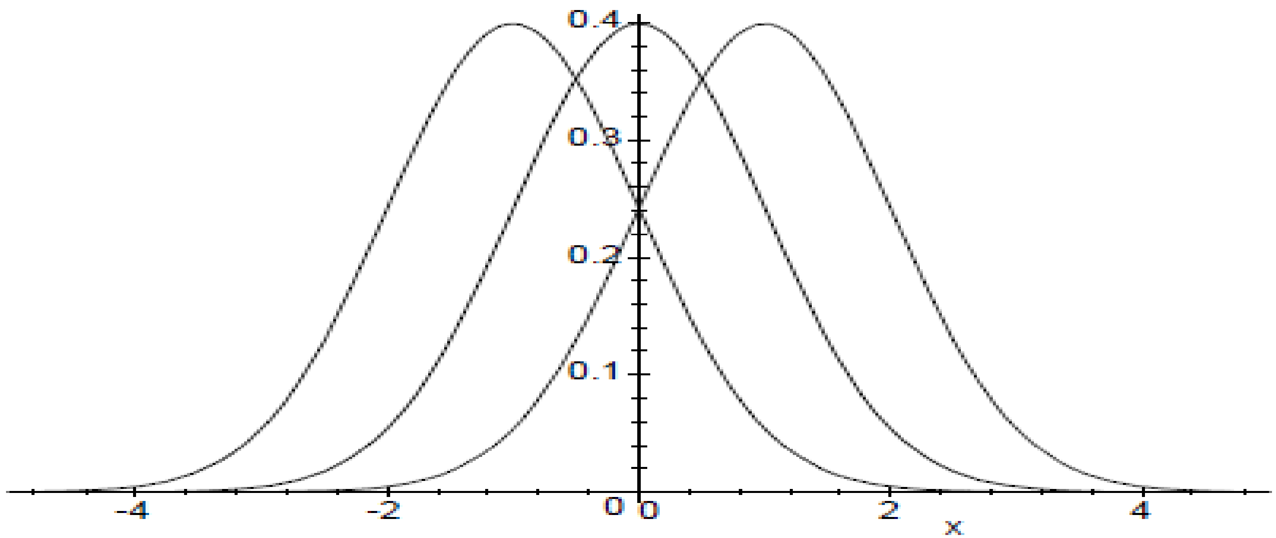
- **Cumulative Distribution Function**

x	*p(x)*
1	*p(x=1)*=1/6
2	*p(x=2)*=1/6
3	*p(x=3)*=1/6
4	*p(x=4)*=1/6
5	*p(x=5)*=1/6
6	*p(x=6)*=1/6
	1.0



Basic concepts of digital communication

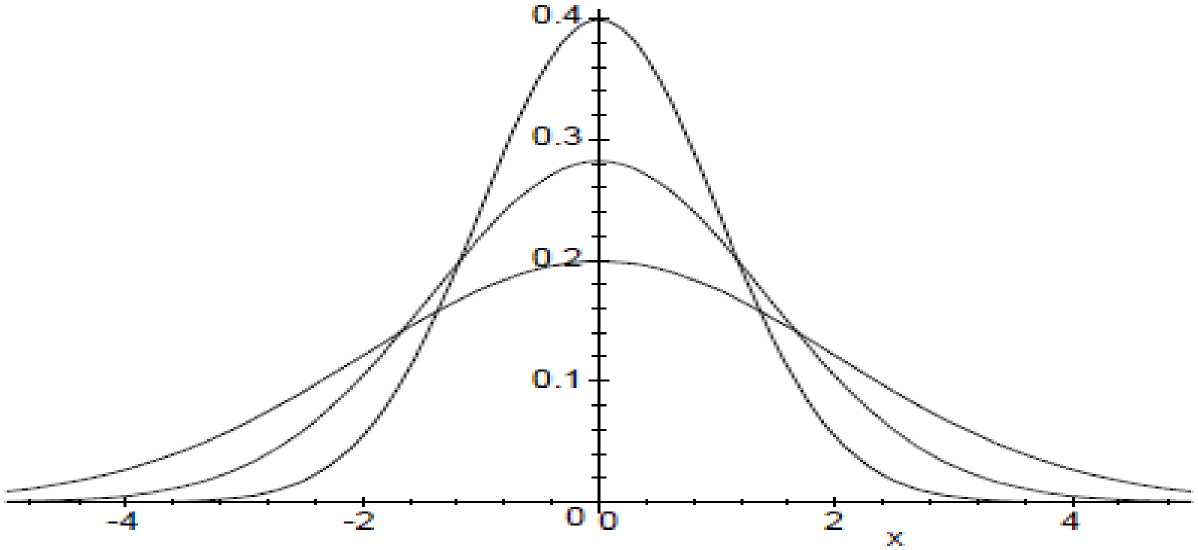
- **Q function**
- **Normal or Gaussian distribution**
- The density of normally distributed variable is bell-shaped.



- Three normal densities with unit variance (σ^2=1) and means μ=-1(dotted), μ=0 (solid), μ=1 (dashed).

Basic concepts of digital communication

•**Normal or Gaussian distribution**



•Three normal densities with zero mean and different variance: σ^2=1 (solid), σ^2=2 (dotted), σ^2=4 dotted.

•Variance (SD) defines the shape (the width of the bell).

Basic concepts of digital communication

- The Gaussian probability function of unit variance and zero mean is

$$Z(x) = \frac{e^{-x^2/2}}{\sqrt{2\pi}}$$

and the corresponding cumulative distribution function is

$$P(x) = \int_{-\infty}^{x} Z(t)dt$$

- The Q-function is defined as

$$Q(x) = 1 - P(x) = \int_{x}^{\infty} Z(t)dt$$

$$Q(x) = \int_{x}^{\infty} \frac{e^{-\mu^2/2}}{\sqrt{2\pi}} du$$

Basic concepts of digital communication

- **Coherent modulation**
- The simplest possible digital communication system is one that transmits a sequence of binary symbols {0 ,1} over a channel with AWGN of spectral density $N_0/2$.
- The two binary symbols are transmitted using two signaling waveforms denoted as $\boldsymbol{s_1(t)}$ and $\boldsymbol{s_2(t)}$.
- These waveforms are defined to exist over the time interval (0, T).
- One of these signals is transmitted each T seconds (Information rate $\boldsymbol{R_b}=1/\boldsymbol{T}$ binary symbols or bits per seconds).
- During signaling interval k, the transmitter associates a symbol, say a 1, with $\boldsymbol{s_1(t-kT)}$ and the other symbol, say a 0 with $\boldsymbol{s_2(t-kT)}$.

Coherent modulation

- The receiver should have a perfect knowledge of both $\boldsymbol{s_1(t)}$ and $\boldsymbol{s_2(t)}$, including:
  - precise time at which they could be received and
  - the probability that they were transmitted
- During each *T*-second signaling interval, the receiver observes the signaling waveform contaminated by AWGN.
- Minimum probability of error (P_E) is achieved when the receiver guesses the transmitted signal to be the signal which, given the received signal plus noise waveform, was most likely to have been transmitted.
- Such a receiver is called a ***maximum-likelihood receiver***.
- Minimum probability of error can be defined as:

$$P_E = Q[\sqrt{z(1-R_{12})}]$$

Basic concepts of digital communication

- **Coherent modulation**
- z is defined as:

$$z = \frac{(E_1 + E_2)/2}{N_0} = \frac{E_b}{N_0}$$

- where E_i, i=1,2, is the energy signal i, defined as:

$$E_i = \int_0^T \left| s_i(t) \right|^2 dt$$

- $E=(E_1+E_2)/2$ is the average signal energy.
- R_{12} is defined as:

$$R_{12} = \frac{\sqrt{E_1 E_2}}{E} \rho_{12}$$

Basic concepts of digital communication

- **Coherent modulation**
- The parameter ρ_{12} , **the normalized correlation coefficient** between two signals, is defined below:

$$\rho_{12} = \frac{1}{\sqrt{E_1 E_2}} \int_0^T s_1(t) s_2(t) dt$$

- If R_{12}=0, signaling scheme is said to be orthogonal, and if R_{12}=-1, scheme is said to be antipodal.
- **Matched filter receiver**
- It consists of the matched filter followed by a sampler, which samples the output of the matched filter at the end of each *T*-second signaling interval, and a threshold comparator.

Two structures of receivers

- **Matched filter receiver**
  - In signal processing, a matched filter is obtained by correlating a known signal or template, with an unknown signal to detect the presence of the template in the unknown signal.
  - This is equivalent to convolving the unknown signal with a conjugated time-reversed version of the template.
  - It consists of the matched filter followed by a sampler, which samples the output of the matched filter at the end of each *T*-seconds signaling interval, and a threshold comparator.

Two structures of receivers

- **Matched filter receiver**

➢For equally probable signals, the comparator threshold is set at

$$k = \frac{1}{2}\left[s_{01}(T) + s_{02}(T)\right]$$

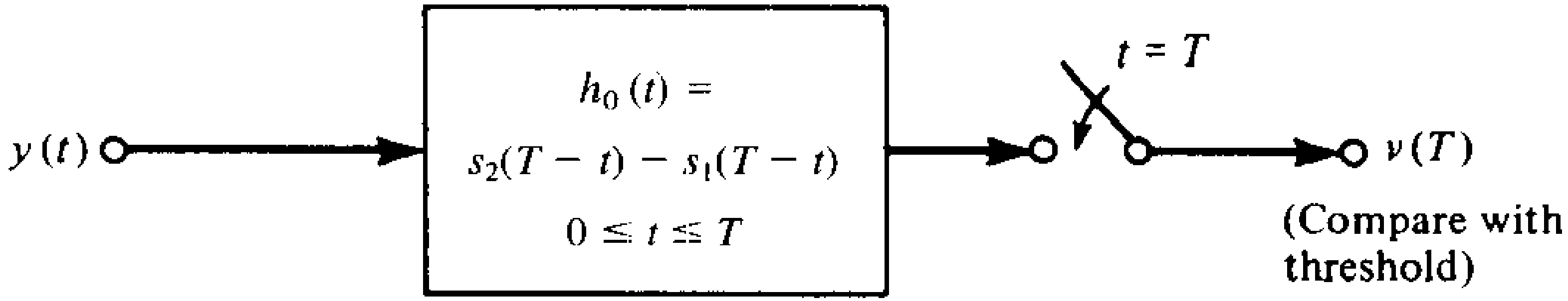
Where $s_{o1}(T)$ and $s_{o2}(T)$ are output signals from a matched filter at the sampling instant, corresponding to $s_1(t)$ and $s_2(t)$, respectively, at its input.

➢A matched filter for any signal has an impulse response that is the shifted time reverse of the signal. Since we are dealing with two signals in this case, the matched filter is matched to the difference of the two signals and has an impulse response:

$$h_0(t) = s_2(T-t) - s_1(T-t)$$

Basic concepts of digital communication

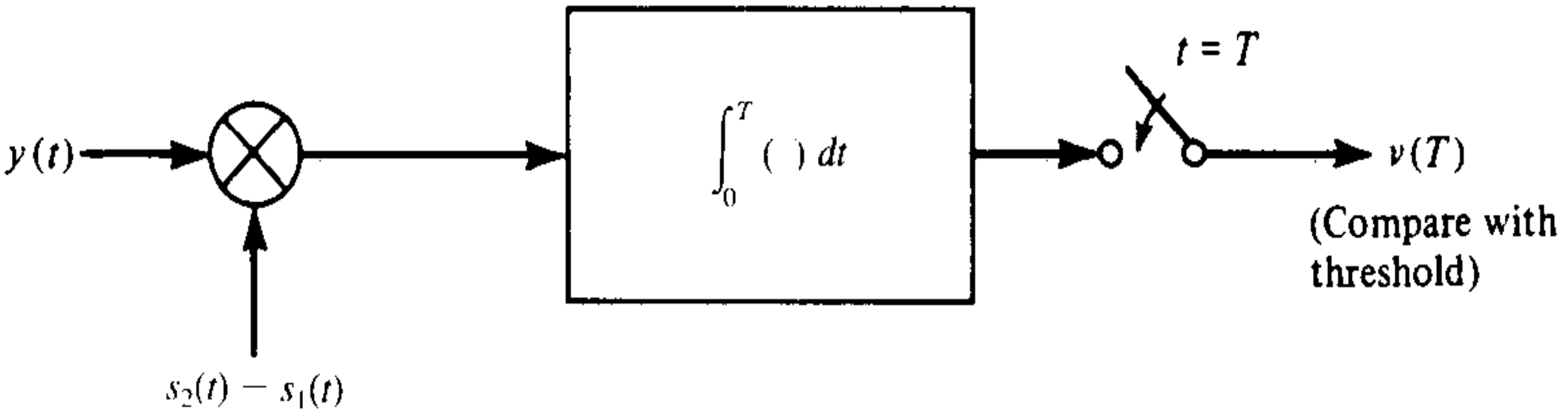
- **Matched filter receiver**
  - The matched filter is the optimal linear filter for maximizing the signal to noise ratio in the presence of additive stochastic noise.
  - Matched filters are commonly used in radar, in which a known signal is sent out, and the reflected signal is examined for common elements of the out-going signal.



(a) Matched filter implementation

Correlator receiver

- It consists of a correlation operation with the difference of the two signals, followed by a sampler and a threshold comparison.
- The matched filter and correlator implementations are equivalent and may be demonstrated by writing down the signal plus noise outputs of each operation at the sampling instant and showing them to be equal.



(b) Correlator implementation

Basic concepts of digital communication

- **Signal space**
- Signals have characteristics that are similar to vectors. To represent signal waveforms, vectors can be used.
- **Vector space concepts**
- A vector **v** in an n-dimensional space is characterized by its n components $[v_1, v_2, \dots, v_n]$. it may also be represented as a linear combination of unit vectors or basis vectors $\mathbf{e}_i$, $1 \le i \le n$, i.e.

$$v = \sum_{i=1}^{n} v_i e_i$$

- where, by definition, a unit vector has length of unity and v_i is the projection of the vector **v** onto the unit vector e_i.

Basic concepts of digital communication

- **Vector space**
- **Inner product of two vectors**
- **Orthogonal vectors**
- Inner product of two n-dimensional vectors $\mathbf{v}_1$=[v_{11}, v_{12}, … , v_{1n}] and $\mathbf{v}_2$=[v_{21}, v_{22}, … , v_{2n}] is defined as:

$$v_1 . v_2 = \sum_{i=1}^{n} v_{1i} v_{2i}$$

- Two vectors are orthogonal if $v_1.v_2=0$.
- Example: u=(3,0) and v=(0,5)
- u.v=3.0+0.5=0

Basic concepts of digital communication

- **Vector space**
- More generally, a set of m vectors v_k, $1 \leq k \leq m$ are orthogonal if

$$v_i . v_j = 0$$

- The norm of a vector v is denoted by || v || and is defined as

$$v = (v.v)^{1/2} = \sqrt{\sum_{i=1}^{n} v_i^2}$$

- which is simply its length.
- **Orthonormal vectors**
- A set of m vectors is said to be orthonormal if the vectors are orthogonal and each vectors has a unit norm.

Basic concepts of digital communication

- **Vector space**
- **Linearly independence**
- A set of m vectors is said to be ***linearly independent*** if no one vector can be represented as a linear combination of the remaining vectors.
- **Triangle inequality**
- Two n-dimensional vectors v_1 and v_2 satisfy the triangle inequality

$$\| v_1 + v_2 \| \leq \| v_1 \| + \| v_2 \|$$

- with equality if v_1 and v_2 are in the same direction, i.e. v_1=a v_2 where a is a positive real scalar.

Basic concepts of digital communication

- **Vector space**
- **Cauchy-Schwartz inequality**
- From triangle inequality, follows Cauchy-Schwartz inequality

$$|v_1.v_2| \leq \|v_1\| \, \|v_2\|$$

- with equality if v_1=a v_2. The norm square of the sum of two vectors may be expressed as

$$\|v_1+v_2\|^2 = \|v_1\|^2 + \|v_2\|^2 + 2v_1.v_2$$

- If v_1 and v_2 are orthogonal then $v_1 . v_2$=0 and hence,

$$\|v_1+v_2\|^2 = \|v_1\|^2 + \|v_2\|^2$$

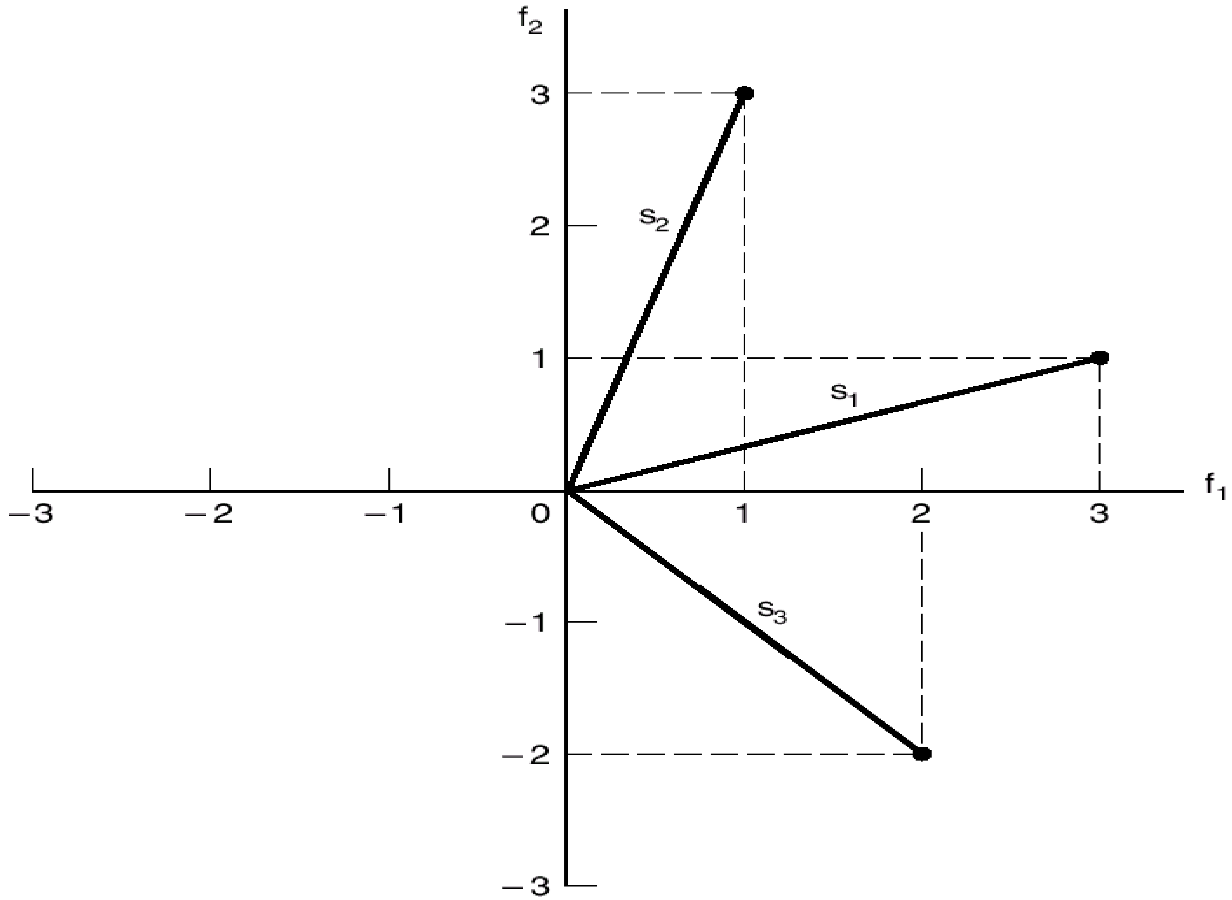
Basic concepts of digital communication

- **Vector space**
- This is the Pythogorean relation for two orthogonal n-dimensional vectors.
- **Gram-Schmidt Procedure**
- By this procedure, a set of n-dimensional vectors v_i, $1 \le i \le m$ is used to construct a set of orthonormal vectors.
- Geometric representation of vectors / signals is achieved with the help of this procedure.
- Consider real value energy signals $s_1(t), s_2(t),..s_M(t)$, each of duration T sec

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\phi_j(t), \qquad \begin{Bmatrix} 0 \le t \le T \\ i==1,2,....,M \end{Bmatrix} \qquad (5.5)$$

Basic concepts of digital communication

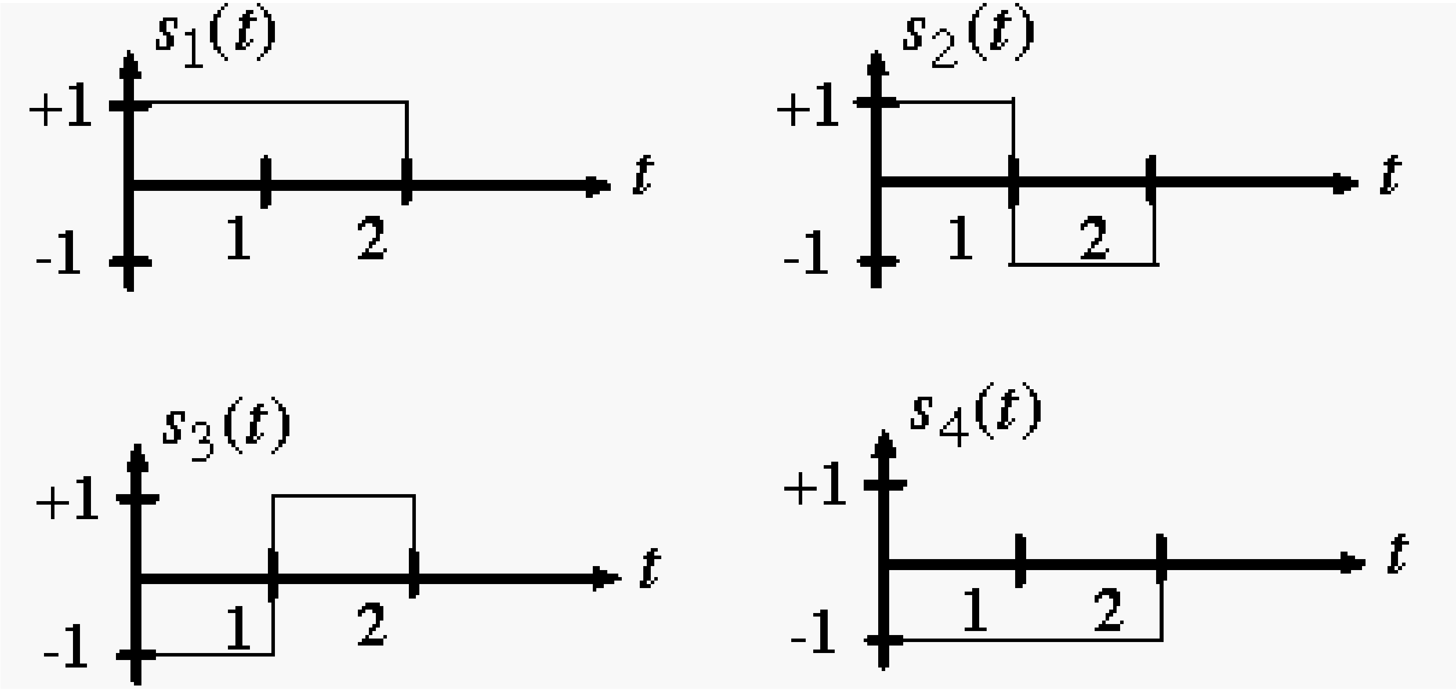
- **Geometric representation of signals (three signals in 2 dimensions)**



Basic concepts of digital communication

- **Example 1**

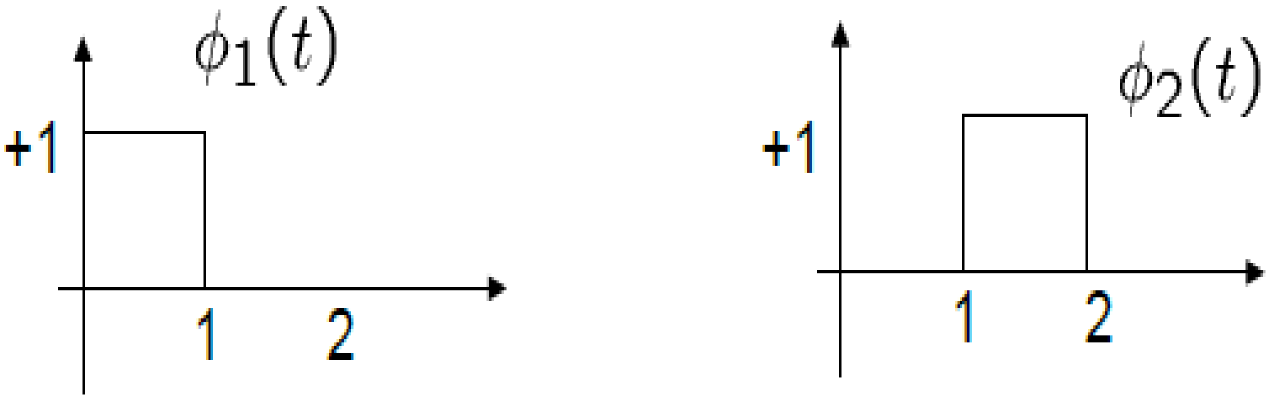
Consider following signal set



Basic concepts of digital communication

- **Example 1**

By inspection, the signals can be expressed in terms of the following two basis functions:



Basic concepts of digital communication

- **Example 1**

$$s_1(t) = 1.\phi_1(t) + 1.\phi_2(t) \qquad s_2(t) = 1.\phi_1(t) - 1.\phi_2(t)$$

$$s_3(t) = -1.\phi_1(t) + 1.\phi_2(t) \qquad s_4(t) = -1.\phi_1(t) - 1.\phi_2(t)$$

- Note that the bases are orthonormal:

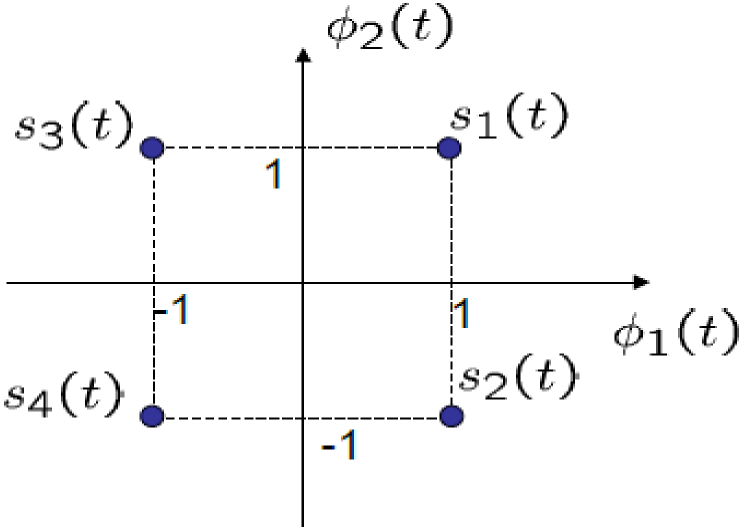
$$\int_{-\infty}^{\infty} \phi_1(t)\phi_2(t)\,dt = 0$$

- Also note that each of these functions has unit energy:

$$\int_{-\infty}^{\infty} |\phi_1(t)|^2\,dt = \int_{-\infty}^{\infty} |\phi_2(t)|^2\,dt = 1$$

Basic concepts of digital communication

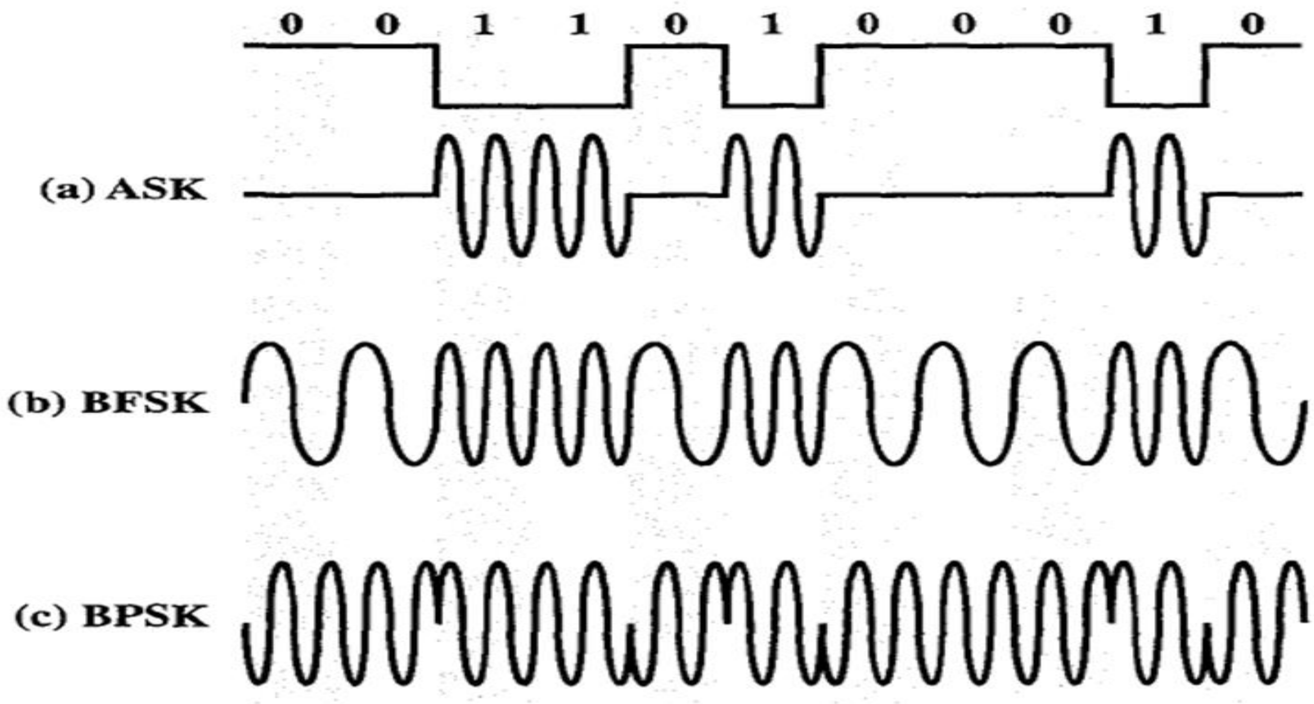
- **Example 1**
- Constellation diagram
- It is a representation of digital modulation scheme in the signal space.
- Axes are labeled as $\phi_1(t)$ and $\phi_2(t)$.



- Possible signals are plotted as points called constellation points.

Basic concepts of digital communication

- **Coherent modulation**
- **Binary Phase Shift Keying (BPSK)**



Basic concepts of digital communication

- **Coherent modulation**
- **Binary Phase Shift Keying (BPSK)**
- If the signal received at the receiver is in phase error ϕ, the probability of error for BPSK is as under:

$$P_E(\phi) = Q\left(\sqrt{2z\cos^2\phi}\right) \qquad \phi \le \pi$$

- If the phase error ϕ is a random variable with probability density function p (ϕ), the average probability of error is

$$\overline{P_E(\phi)} = \int_{-\pi}^{\pi} P_E(\phi)\, p(\phi)\, d\phi$$

Basic concepts of digital communication

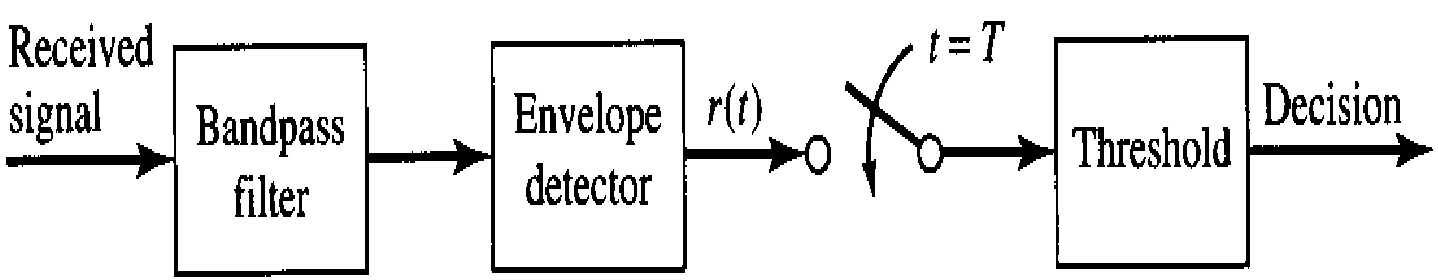
•**Noncoherent modulation**

• In situations, where phase can not be stable, (e.g. fading channels), it is useful to employ modulation schemes that do not require acquisition of a reference signal at the receiver which is in phase coherence with the received signal.

•ASK and FSK are examples of non-coherent modulation

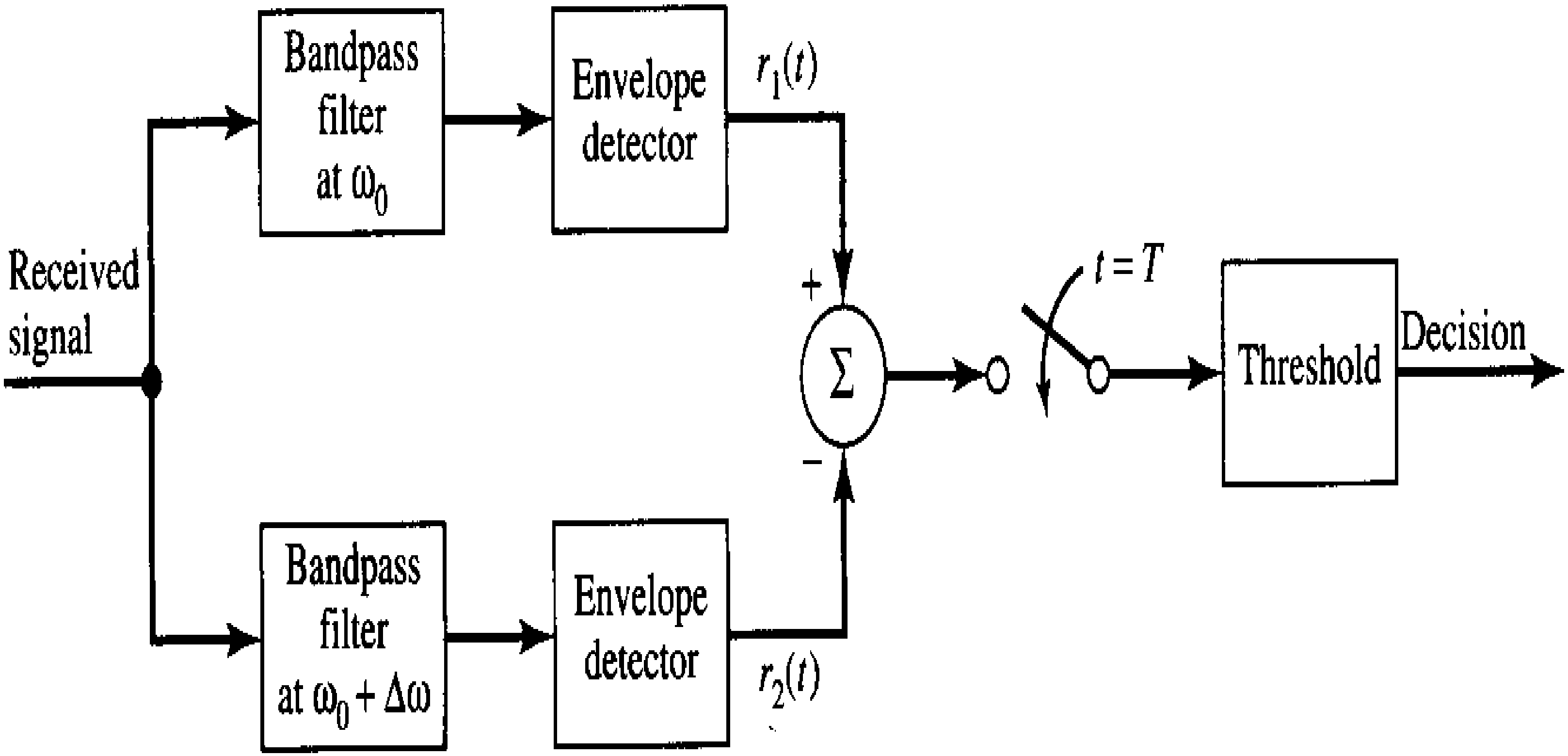
•Receivers for both schemes are shown below:

•ASK



Basic concepts of digital communication

- **Noncoherent modulation**
- FSK



Basic concepts of digital communication

- **Noncoherent ASK**
- P_E for large signal-to noise ratios can be approximated as:

$$P_E \cong \frac{1}{2} e^{-z/2} \quad z \gg 1$$

- **Noncoherent FSK**
- For noncoherent detection of FSK, P_E is given by:

$$P_E = \frac{1}{2} e^{-z/2}$$

- **Differential PSK (DPSK)**
- In this scheme, the phase of the preceding bit interval is used as a reference for the current bit interval .
- This technique depends on the channel having enough phase stability so that phase changes are minimum due to channel disturbance .

Basic concepts of digital communication

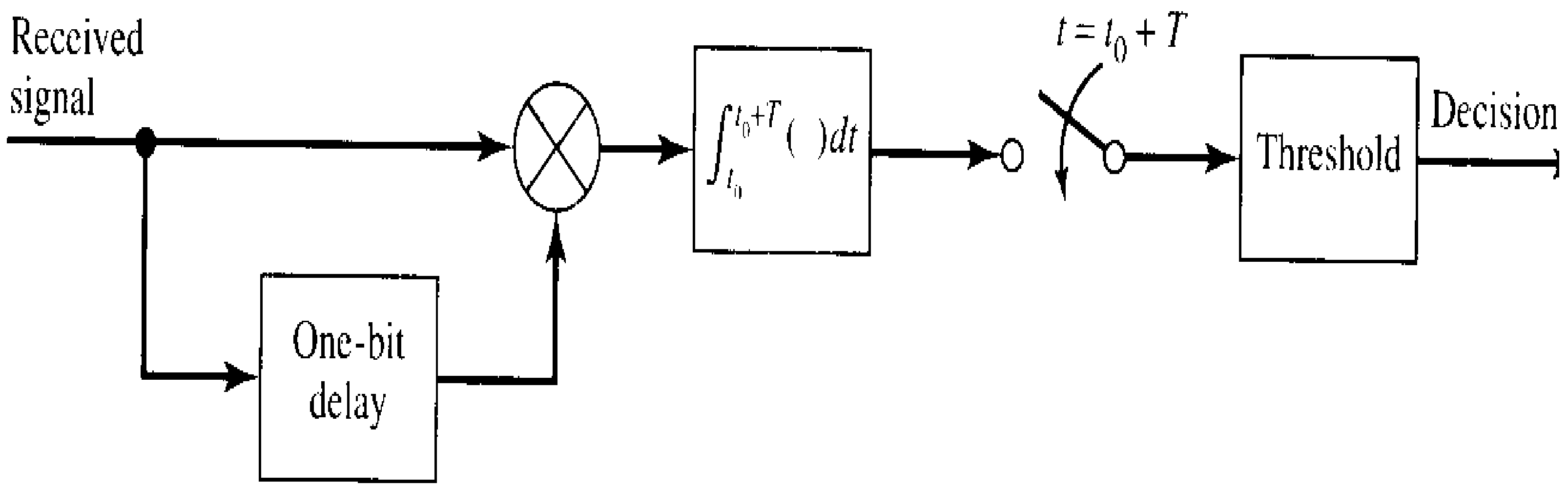
- **Noncoherent modulation**
- **Differential encoding**
- This technique also depends on there being known phase relationship from one bit interval to the next bit interval.
- It is therefore called differentially coherent scheme.

Message Sequence		1	0	0	1	1	1	0
Encoded Sequence	1	1	0	1	1	1	1	0
Transmitted Phase (rad)	0	0	π	0	0	0	0	π

- A block diagram is shown on next slide.

Basic concepts of digital communication

- **Noncoherent modulation**
- **Differential encoding**



- P_E is given below:

$$P_E = \frac{1}{2} e^{-z}$$

Basic concepts of digital communication

- **Performance comparison**